

RELATION AND MAPPINGS

Summary:

A relation between members of the given set can be illustrated using a papygram

EXAMPLES:

- 1. Draw a papygram illustrating the relation "is a prime factor of" in the set $\{1, 2, 3, 4, 5, 6, 8, 12, 30\}$*
- 2. Draw a papygram showing the relation "is a multiple of" in the set $\{42, 28, 21, 14, 7\}$*
- 3. Given that $T = \{2, 5, 6, 8, 9, 10, 12, 13\}$, illustrate on papygrams the relations: (i) "Greater than by 3"
(ii) "Factor of"*

FUNCTIONS

Summary:

- 1. A function $f(x)$ is a formula in terms of x .*
- 2. A function f that maps x on to $3x + 1$ can either be written as follows:*

(i) $f(x) = 3x + 1$ *(ii) $f: x \rightarrow 3x + 1$* *(iii) $x \rightarrow 3x + 1$*
- 3. (i) A mapping diagram is an arrow diagram with a set of values of x and that
of $f(x)$.*
(ii) A set of values of x is called The Domain
(iii) A set of values of $f(x)$ is called The Range
- 4. The inverse of a function $f(x)$ is denoted by $f^{-1}(x)$. This function maps the
range back on to the domain.*

5. A function is undefined or meaningless if its denominator part is equal to zero.

6. (i) A composite function $fg(x)$ is a combination of two functions $f(x)$ and $g(x)$.

(ii) A composite function $f^2(x)$ is the same as $ff(x)$.

EXAMPLES:

1. (i) Determine the range corresponding to the domain $\{0, 1, 2, 3\}$ for the mapping $f(x) = 3x + 1$.

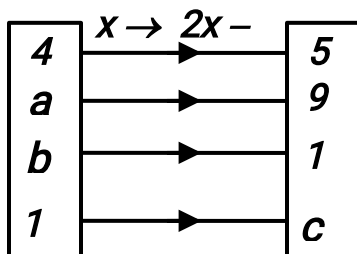
(ii) Represent the mapping in (i) above on an arrow diagram.

2. (i) Determine the range corresponding to the domain $\{-3, -2, 0, 1, 2, 3\}$ for the

mapping $x \rightarrow x^2 + 1$.

(ii) Represent the mapping in (i) above on an arrow diagram.

3. Find the unknown values in the arrow diagram for the mapping $x \rightarrow 2x - 3$.



4. Given the function $f(x) = \frac{10x}{x^2 - 4}$, find:

(i) $f(3)$

(ii) $f(6)$

(iii) the values of x for which $f(x)$ is undefined

(iv) the values of x for which $f(x) = 6$

5. Given the function $f(x) = \frac{2x}{3x^2 - 10x - 8}$, find:

(i) $f(2)$

(ii) $f(-1)$

(iii) the values of x for which $f(x)$ is undefined

6. Given that $f(x) = 3x + 5$, $g(x) = \frac{2}{2x - 6}$ and $h(x) = \frac{4 - x^2}{x^2 + 3}$, find:

(i) $f^{-1}(x)$, and hence $f^{-1}(-1)$

(ii) $g^{-1}(x)$, and hence $g^{-1}(-2)$

(iii) $h^{-1}(x)$, and hence $h^{-1}(0)$

7. Given that $f(x) = ax - 7$ and $f(8) = 17$, find:

(i) the value of a

(ii) $f(4)$

(ii) $f^{-1}(x)$, hence obtain $f^{-1}(8)$.

8. Given that $f(x) = a + bx$, $f(1) = 8$ and $f(-1) = 2$, find:

(i) the values of a and b

(ii) $f(-2)$

(iii) $f(5)$

(iv) the value of c for which $f(c) = -7$

(v) $f^{-1}(x)$

(vi) $f^{-1}(-7)$

10. Given that $f(x) = ax + 9$ and $f^{-1}(13) = 1$, find:

(i) the value of a

(ii) $f^{-1}(1)$

11. Given that $f(x) = px + 7$ and $g^{-1}(x) = \frac{5-2x}{3}$, find:

(i) $g(x)$

(ii) the value of p for which $g(2x - 3) = f(x)$

12. Given that $g^{-1}(x) = \frac{1+x}{x}$, find $g(3)$.

13. Given that $f(x) = \frac{2}{x+2} + \frac{8x+4}{x^2-4}$, express $f(x)$ in the form

$$\frac{ax}{x^2 + b}.$$

Hence find:

(i) $f(3)$

(ii) the values of x for which $f(x)$ is undefined

14. Given that $f(x) = \frac{2}{3x + 2} + \frac{5x + 3}{9x^2 - 4}$, express $f(x)$ in the form $\frac{ax + b}{cx^2 + d}$.

Hence find:

(i) $f(2)$

(ii) the values of x for which $f(x)$ is undefined

15. Given that $f(x) = x^2 - 12$ and $g(x) = 2x - 5$, find:

(i) an expression for $gf(x)$ and hence evaluate $gf(4)$

(ii) an expression for $fg(x)$ and hence evaluate $fg(2)$

(iii) the values of x for which $gf(x) = fg(x)$

(iv) an expression for $gg(x)$ and hence evaluate $gg(2)$

(v) an expression for $ff(x)$ and hence evaluate $ff(-3)$

16. Given that $f(x) = 5x - 7$, find $g(x)$ for which:

(i) $fg(x) = 10x + 8$, hence evaluate $g(4)$

(ii) $fg(x) = 20x^2 - 37$, hence evaluate $g(2)$

17. Given that $f(x) = x^2 - 7$ and $g(x) = x + 1$, find the values of x for which

$$fg(x) + gf(x) = 0.$$

18. Given that $f(x) = \frac{x+3}{2}$ and $g(x) = \frac{1-2x}{5}$, find the values of x for

$$\text{which } fg(x) = \frac{8x^2 + 24x + 9}{10}.$$

19. Given that $f(x) = \frac{2}{2x-6}$ and $g(x) = x^2 - 1$, find the values of x for

which $fg(x)$ is undefined

EER:

1. Given that $f(x) = \frac{14x}{x^2 - 9}$, find:

(i) $f(4)$

(ii) the values of x for which $f(x)$ is undefined.

2. (i) Determine the range corresponding to the domain $\{4, 9, 16\}$ for the mapping

$$x \rightarrow 10 - 2\sqrt{x}$$

(ii) Represent the mapping in (i) above on an arrow diagram.

3. Express $x^2 + 7x + 12$ in the form $(x + a)^2 + b$. Hence solve the equation

$$x^2 + 7x + 12 = 0.$$

4. Given that $f(x) = 2x + 4$ and $g(x) = x + 5$, find $fg(x)$ and hence evaluate $fg(4)$

5. Given that $f(x) = (x - 3)^2$, find:

(i) $f(5)$

(ii) the values of x for which $f(x) = 16$.

6. Given that $f(x) = ax + 3$ and $f(5) = 33$, find:

(i) the value of a

(ii) $f(-2)$

7. Given that $f(x) = ax^2 + b$, $f(-2) = 3$ and $f(1) = -2$, find:

(i) the values of a and b

(ii) $f(4)$

8. Given that $f(x) = 2x - 5$, find:

(i) $f(-2)$

(ii) $f^{-1}(x)$

9. Given that $f(x) = 3x - 4$, find $f^{-1}(5)$.

10. Given that $f(x) = \frac{1}{2}(x + 7)$ and $g(x) = x(x - 6)$, find the values of x for

which

which $f^{-1}(x) = g(x)$.

11. Given that $f(x) = \frac{2}{x+4} + \frac{4}{x-3} - \frac{4(x+4)}{x^2+x-12}$, express $f(x)$ in the

form $\frac{a}{x+b}$. Hence evaluate $f(-3)$

12. Given that $f(x) = \frac{7}{x+3} + \frac{7x+21}{x^2-9}$, express $f(x)$ in the form $\frac{ax}{x^2+b}$.

Hence find:

(i) $f(4)$

(ii) the values of x for which $f(x)$ is undefined

13. Given that $f(x) = ax^2 + bx$, $f(1) = 5$ and $f(2) = 14$, find:

(i) the values of a and b

(ii) $f(3)$

14. Express $x^2 + x - 12$ in the form $(x+a)^2 + b$. Hence solve the equation

$$x^2 + x - 12 = 0.$$

15. Express $3x^2 + 4x - 4$ in the form $a(x+b)^2 + c$. Hence solve the

equation

$$3x^2 + 4x - 4 = 0.$$

16. Given that $f(x) = \frac{3}{1 - x^2}$, find the values of x for which $f(x) = 4$.

17. Given the function $f(x) = \frac{8x}{3x^2 + 9x - 30}$, find:

(i) $f(-1)$

(ii) the values of x for which $f(x) = 1$

(iii) the values of x for which $f(x)$ is undefined

18. Given that $f^{-1}(x) = \frac{\beta x - 2}{\beta + 4x}$ and $f(2) = -4$, find the value of β . Hence find

the value of x for which $f(x)$ is undefined.